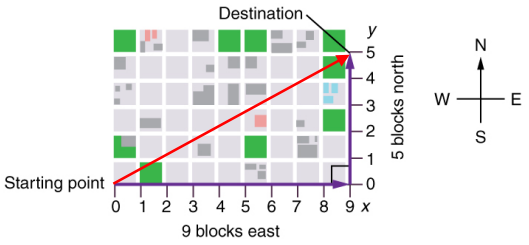
Two Dimensional Motion

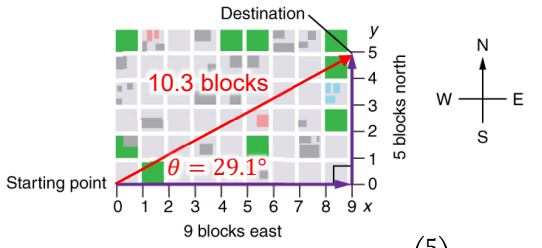
- A person walks a two-dimensional path between two points.



- The resulting displacement is the straight-line path between the two points.

OpenStax, Rice University (CC BY 4.0)

- The resulting displacement vector can be calculated using the Pythagorean theorem and trigonometry.

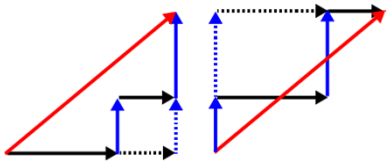


$\sqrt{9^2 + 5^2} = 10.3$ $\theta = \tan^{-1}\left(\frac{5}{9}\right) = 29.1°$

OpenStax, Rice University (CC BY 4.0)

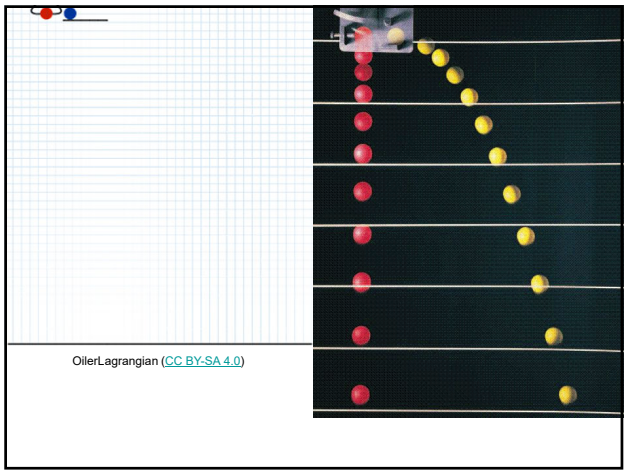
Independence of Motion

- The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

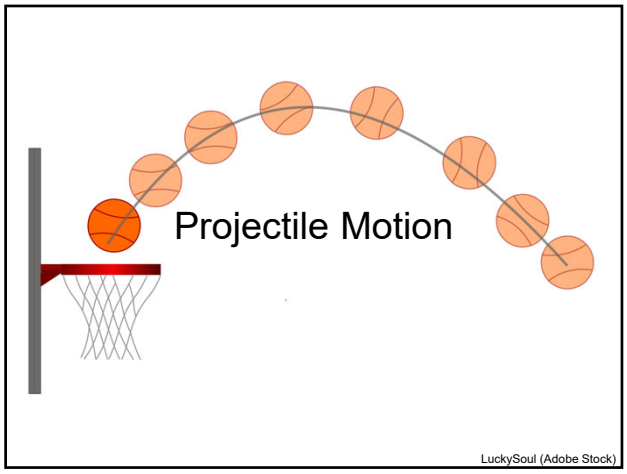


Waifer X (CC BY 2.0)

https://youtu.be/zMF4CD7i3hg



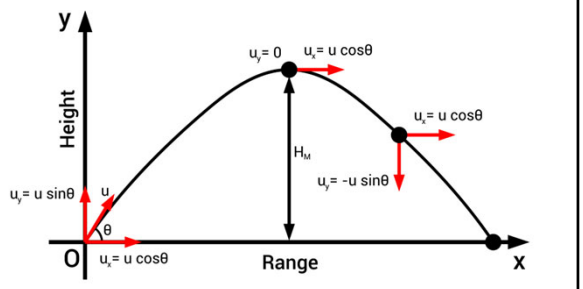
OilerLagrangian (CC BY-SA 4.0)



LuckySoul (Adobe Stock)

- When an object travels through the air, the vertical motion can be separated from the horizontal motion.
- Gravity affects the vertical motion of the object causing it to accelerate in the vertical direction.

	Horizontal	**Vertical**
Acceleration	0	g ($9.8\ m/s^2$) down
Velocity	Constant	Changing



Zizo (Adobe Stock)

Example 1

- A cannon ball is launched with a horizontal velocity of 50 m/s from the top of a 10 m high cliff. Calculate the distance from the bottom of the cliff where the cannon ball lands.

Separate the horizontal and vertical velocities.

- Horizontal
- $v_{xi} = 50\ \text{m/s}$
- $a_x = 0$
- $x = ?$
- $t = ?$

- Vertical
- $v_{yi} = 0$
- $a_y = g = -9.8\ \text{m/s}^2$
- $y = -10\ \text{m}$
- $t = ?$

Solve for time, t, vertically.

- Vertical

$$d_y = v_{yi}t + \frac{1}{2}a_y t^2$$

$$d_y = \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2d_y}{g}}$$

$$t = \sqrt{\frac{2(-10)}{-9.8}} = 1.43 \text{ s}$$

- The time it takes for the object to fall and hit the ground is the same as the horizontal time.
  - The object stops moving horizontally once the object has hit the ground.
- That means that we can now solve for the horizontal distance.

- Horizontal

$$d_x = v_{xi}t + \frac{1}{2}a_x t^2$$

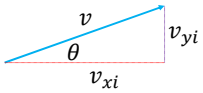
$$d_x = v_{xi}t$$

$$d_x = (50)(1.43) = 71.5 \text{ m}$$

Example 2

- A cannon ball is launched with a velocity of 50 m/s at an angle of 30° from the horizontal from the top of a 10 m high cliff. Calculate the distance from the bottom of the cliff where the cannon ball lands.

Separate the horizontal and vertical velocities.



- Horizontal
- $v_{xi} = 50\cos 30 \text{ m/s}$
- $a_x = 0$
- $x = ?$
- $t = ?$

- Vertical
- $v_{yi} = 50\sin 30 \text{ m/s}$
- $a_y = -g = -9.8 \text{ m/s}^2$
- $y = -10 \text{ m}$
- $t = ?$

Solve for time, *t*, vertically.

- vertical

$$d_y = v_{yi}t + \frac{1}{2}a_y t^2$$

$$-10 = (50\sin 30)t + \frac{1}{2}(-9.8)t^2$$

$$4.9t^2 - 25t - 10 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(4.9)(-10)}}{2(4.9)}$$

$$t = \begin{cases} \cancel{-0.37 \text{ s}} \\ 5.47 \text{ s} \end{cases}$$

- Horizontal

$$d_x = v_{xi}t + \frac{1}{2}a_x t^2$$

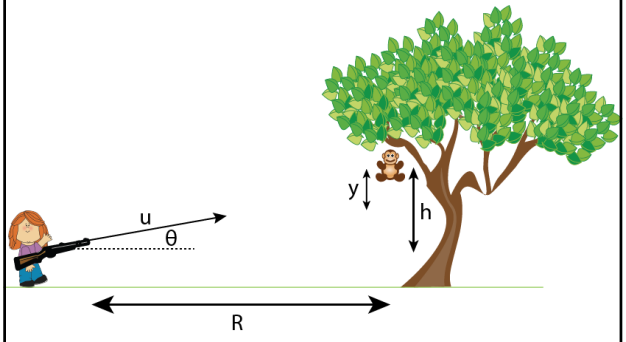
$$d_x = v_{xi}t$$

$$d_x = (50\cos 30)(5.47) = 237 \text{ m}$$

The Monkey and the Hunter

Girl: © MyCuteGraphics.com. Used with permission.
Monkey: public domain (openclipart.com)

A hunter with a gun goes out in the woods to hunt for monkeys and sees one hanging in a tree. The monkey releases its grip the instant it hears the gun. Where should the hunter aim to hit the monkey?



Girl: © MyCuteGraphics.com. Used with permission.
Gun: Firkin – public domain (openclipart.com)
Tree: public domain (openclipart.com)
Monkey: public domain (openclipart.com)

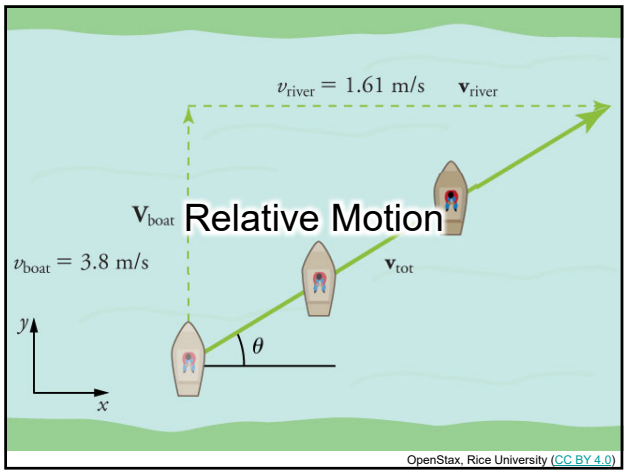
Mechanics–
Projectile Motion
Monkey and Hunter

Monkey and a Gun

MIT Department of Physics
Technical Services Group

https://youtu.be/CTSwbyCAGHU?t=368

https://youtu.be/cxvsHNRXLjw



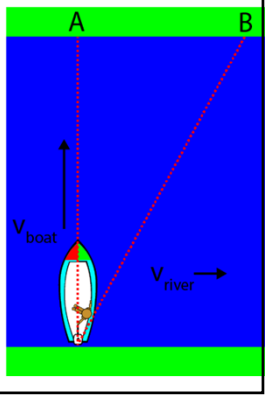
- If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves diagonally relative to the shore, because the river carries the boat downstream.
- The boat has a velocity relative to a river and the river has a velocity relative to an observer on solid ground.
- The velocity of the boat relative to the observer is the sum of these velocity vectors.

Example

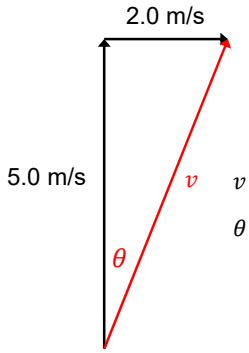
A boat with a velocity of 5.0 m/s is crossing a 50.0 m wide river with a current of 2.0 m/s towards the east.

a) What is the velocity of the boat relative to the shore?

b) What is the distance from A to B?



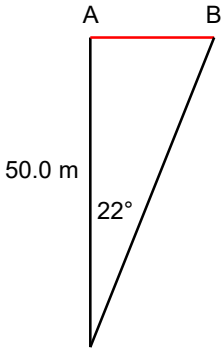
a) Velocity relative to the shore



$v = \sqrt{5^2 + 2^2} = 5.4 \text{ m/s}$

$\theta = \tan^{-1}\left(\frac{2}{5}\right) = 22°$ east of north

b) Distance from A to B



$d = 50 \tan 22 = 20.2 \text{ m}$